

**SAINF:**  
**A Toolbox for Analyzing the Effect of**  
**Point-like, Line-like and Polygon-like Infrastructural Features**  
**on the Distribution of Point-like Non-infrastructural Features**

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## **PREFACE**

This manual describes how to use SAINF: a toolbox for analyzing the effect of infrastructural features on the distribution of point-like non-infrastructural features. SAINF is part of the results obtained from the six year (1998-2003) project entitled “Spatial Information Science for Human and Social Sciences” funded by the Grant-In-Aid for Special Field Studies B, the Ministry of Education and Science, Japan. The leader was Atsuyuki Okabe.

## **ACKNOWLEDGEMENTS**

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## NOTICE

The authors distribute SAINF only to those who agree on the following conditions.

1. The user will use SAINF for nonprofit purposes only.
2. The authors will not bear responsibility for any trouble that the user may meet in the use of SAINF.
3. When the user uses SAINF, he/she will report to the authors his/her name, affiliation, address and e-mail address.
4. When the user publishes any results obtained by using SAINF, he/she will explicitly state in the paper that he/she used SAINF. Also, he/she will send a reprint of the paper to the authors.
5. The user will report to the authors any trouble he/she meets in the use of SAINF. (The authors will remove bugs, if any, at their earliest convenience.)

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## 1. Introduction

In the real world we notice various kinds of phenomena such that the distribution of one type of features is affected by the configuration of another type of features. For example, fast-food stores tend to locate around stations; drugstores tend to locate around hospitals; boutiques tend to locate along the main street; textiles factories tend to locate near a river; high-class apartment buildings tend to locate around big parks; lodging buildings for students tend to locate around university campuses. The objective of this paper is to introduce a user-friendly GIS-based toolbox, called SAINF, for analyzing these locational trends statistically.

About two decades ago, when researchers wanted to use quantitative methods for spatial analysis, they had to develop computer programs by themselves. As a result, the use of those methods was very limited. This limitation was resolved by the emergence of user-friendly GIS in the late 1980's. To incorporate this new technology into spatial analysis, methodologies for linking spatial analysis to GIS were discussed (Goodchild, 1987; Anselin and Getis, 1992; Anselin et al. 1992; Goodchild et al. 1992; Fotheringham and Rogerson, 1994), and GIS-based toolboxes for spatial analysis began to develop (Walker and Moor, 1988; Haslett et al. 1990; Openshaw et al. 1990; Openshaw et al. 1991). At present many GIS-based toolboxes for spatial analysis are available (Getis, 2000; Volume 2, No.1 of Journal of Geographical Systems), and they are found at

<http://www.csiss.org/clearinghouse/> (Center for Spatially Integrated Social Science);

<http://ua.t.u-tokyo.ac.jp/okabelab/satools/>.

The former site provides a powerful search engine for spatial tools, and the latter site provides a web system for finding sites serving free software packages for spatial analysis (the outline is shown in Itoh and Okabe (2003)).

Although many tools are found on the web space, they do not completely cover methods for spatial analysis. The toolbox SAINF provides a tool for spatial analysis to fill this gap. To introduce this tool explicitly, let us refer to two classifications. First, we classify features into two types by the duration of life: 'non-infrastructurel' features or 'infrastructurel' features. The non-infrastructurel features mean the features that usually last for a few decades or so. As referred to in the first paragraph, examples are: fast-food stores, drugstores, boutiques, and apartment buildings. The infrastructurel features mean the features that last for one hundred years or more. Examples are:

stations, main streets, and big parks that are referred to in the first paragraph. The toolbox SAINF is particularly useful for examining the effect of infrastructural features on the distribution of non-infrastructural features. Considering this nature, we name our toolbox SAINF (Spatial Analysis for the effect of INFrastructural features on the distribution of non-infrastructural features). Note that we may also use SAINF when distinction between non-infrastructure and infrastructure is not always clear (the effects of two classes of features are bilateral, or the direction of the effects is unknown). In this case, we may examine the effect in two directions: the effect of A features on the distribution of B features and vice versa.

Second, we classify features into three types by size and shape: point-like features, line-like features and polygon-like features. One of the most distinctive characteristics of SAINF is that it can deal with the effect of not only point-like infrastructural features but also line-like and polygon-like infrastructural features on the distribution of non-infrastructural features. Note that since non-infrastructural features are mostly point-like features, SAINF assumes that non-infrastructural features are point-like features.

## 2. Statistical Methods Used in SAINF

SAINF employs two statistical methods: the conditional nearest neighbor distance method (Okabe and Miki, 1984; Okabe and Fujii, 1994; Okabe et al., 1988; Okabe and Yoshikawa, 1989), and the cross  $K$ -function method (Ripley, 1981). The conditional nearest neighbor distance method is different from the ordinary nearest neighbor distance method (Dacey, 1968), and the  $K$ -function method in SAINF is slightly different from the ordinary one with respect to the boundary treatment. To show these differences and for the ease of introducing SAINF, we briefly refer to these methods in this section.

SAINF considers a bounded polygon,  $S$ , (representing a region) in which a set,  $O = \{o_1, \dots, o_n\}$ , of geometrical objects (representing infrastructural features), and a set,  $P = \{p_1, \dots, p_m\}$ , of points (representing non-infrastructural features) are placed. The geometrical objects in  $O$  may be points, line segments or polygons. SAINF is designed to test the following hypothesis.

**Hypothesis**  $H_o$ : points  $P$  are uniformly and randomly distributed over  $S \setminus O$ . (Note that objects in  $O$  are not necessarily uniformly and randomly distributed on  $S$ ; rather, usually they are non-uniformly distributed.)

The conditional nearest neighbor distance method has two tests: the average neighbor distance test and the goodness-of-fit test. The former method uses the *average nearest neighbor distance*,  $\bar{d}$ , defined by

$$\bar{d} = \frac{1}{m} \sum_{i=1}^m d(p_i, O), \quad (1)$$

where  $d(p_i, O) = \min_{j=1, \dots, n} \{ \min_{q \in o_j} \{ d(p_i, q) \} \}$ , i.e. the distance from  $p_i$  to the nearest point in  $O$ . To use  $\bar{d}$  as a test statistic, we need the probability density function,  $g(\bar{d})$ , of  $\bar{d}$ . In theory, this function is exactly obtained from the function  $F(t)$  which indicates the area of the buffer ring of  $O$  whose width is  $t$ . The derivation of  $F(t)$  is shown in Section 8.3.1 of Okabe et al. (2000). Once we obtain the probability density function  $g(\bar{d})$ , we can exactly test the hypothesis  $H_o$ .

Alternatively, when  $m$  is large, we may approximate the probability density function  $g(\bar{d})$  by the normal distribution according to the central limit theorem, and we can use the ordinary mean test. To be explicit, the test statistic,  $z$ , is given by

$$z = \frac{\bar{d} - \eta}{\frac{\sigma}{\sqrt{m}}}. \quad (2)$$

This follows the standard normal distribution.

The second test of the conditional nearest neighbor distance method is the goodness-of-fit test defined by

$$\chi^2 = \sum_{i=0}^k \frac{(N_i - mQ_i)^2}{mQ_i}, \quad (3)$$

where  $Q_i$  is the probability that the distance from a point-like non-infrastructure feature to its nearest neighbor infrastructure feature is in between  $a_i$  and  $a_{i+1}$  under the hypothesis  $H_0$ , and  $N_i$  is the observed number (frequency) of the points of  $P$  whose nearest neighbor distances are in between  $a_i$  and  $a_{i+1}$ .

SAINF also provides the cross  $K$ -function method for the point set  $O$ . Let  $K_i(t)$  be the number of points in  $P$  whose nearest neighbor distance is less than or equal to  $t$  from the  $i$ th point in  $O$  divided by the total number of points. Then the cross  $K$ -function  $K(t)$  is given by

$$K(t) = \frac{1}{n} \sum_{i=1}^n K_i(t). \quad (4)$$

We can obtain the observed cross  $K$ -function  $K(t)$  from the method used in deriving  $F(t)$ . Therefore we do not have to use the adjusting factor for the boundary effect. The boundary effect is exactly taken into account. Comparing the observed cross  $K$ -function  $K(t)$  and the expected cross  $K$ -function  $E(K(t))$ , we can test the hypothesis  $H_0$ .

### **3. System requirements**

Windows NT, 2000, (XP)

ArcView 8.2 or higher

#### **4. Data Setting**

SAINF uses three types of layers that are formatted in Shape files.

(1) The layer of infrastructural features.

Point-like infrastructural features such as stations are represented by points;

line-like infrastructural features such as streets are represented by lines;

polygon-like infrastructural features such as parks are represented by polygons.

(2) The layer of non-infrastructural features represented by points.

(3) The layer of a study area represented by a polygon.

For illustrative purposes, an actual example is used in which infrastructural features are parks, non-infrastructural features are high-class apartment buildings (so called “mansion” in Japanese), and the study region is the Kiba region in Tokyo.

## **5. Installation of the SAINF software**

Obtain “ToolTemplate.mxd” (an empty ArcMap Document File including SAINF) from the SAINF website and make its copy on your Windows PC where ArcView is installed:

Start ArcMap by double clicking the copy of ToolTemplate.mxd and add the three layers (data (1), (2) and (3) in Section 4) to the ArcMap Document File using the AddData command.

A set of tools in SAINF is indicated by [SAINF Tools] on the left end of the ArcMap menu bar at the top line of the window. Tools in this set are also shown by icons in the sub-menu bar [SAINF Tools] placed on the window. You can start SAINF commands by clicking one of these icons.

## 6. Conditional Nearest Neighbor Distance Method

### 6.1 Creation of buffer rings and trimming off the rings outside the study region

We are going to generate the buffer rings of infrastructural facilities (parks in our example) and trim off the rings outside the study area.

#### 6.1.1 Creation of buffer rings

We use one of the functions of ArcView, [buffer generation].

Open [Tools] in the ArcMap menu.

Click [Buffer Wizard] (Figure 6.1.1).

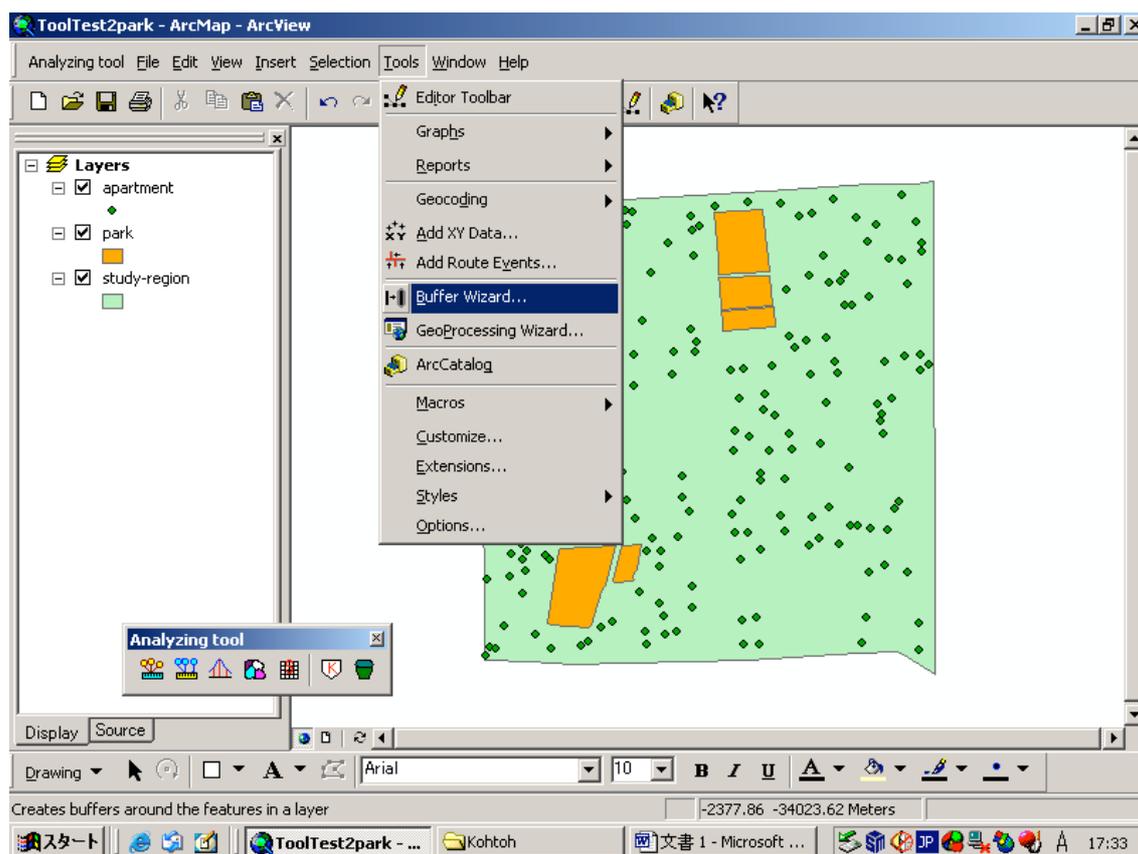


Figure 6.1.1

Then the dialog box shown in Figure 6.1.2 appears.

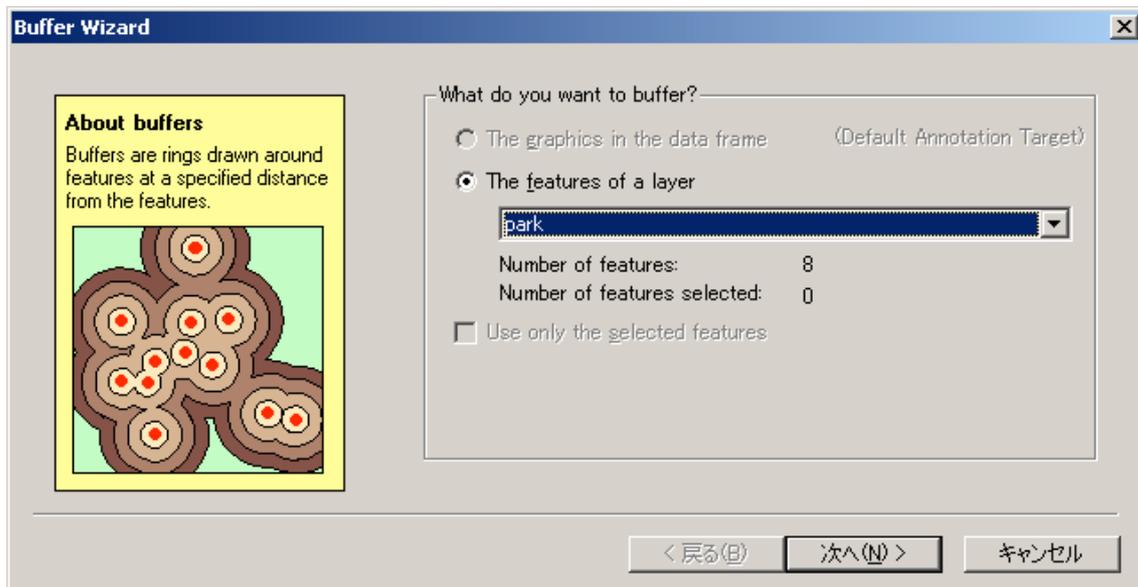


Figure 6.1.2

Click [The features of a layer].

Click the layer of infrastructural features from the pull-down menu. In our case, the layer of infrastructural features is “parks”.

Click [Next].

Then the dialog box shown in Figure 6.1.3 appears.

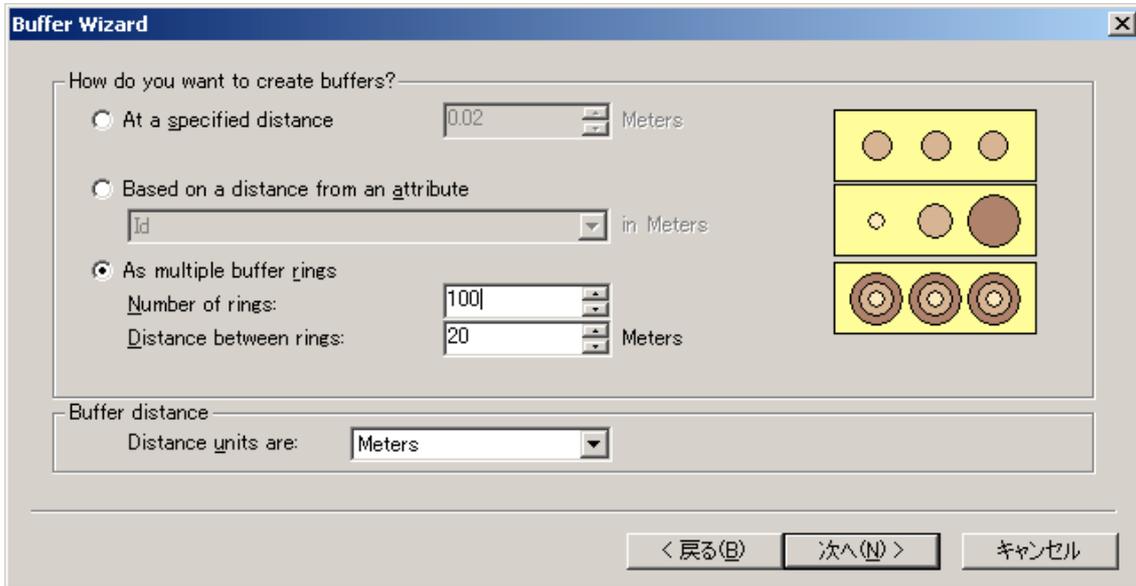


Figure 6.1.3

Check [As multiple buffer rings].

Type the [Number of rings] and [Distance between rings] considering the following factors.

Our experiences suggest that [Number of rings] is around 200. As the number of rings increases, the level of precision increases, but it takes much time to generate buffer rings. In particular, when the infrastructural features are such as streets, the number of elements is large and it takes much time. We may start with 10 rings and increase the number considering the time spent in generating the rings.

The number of buffer rings multiplied by the width of a buffer ring gives the maximum buffer distance. We should determine this maximum buffer distance,  $D_{BFmax}$ , in such a way that the distance,  $D_{Farthest}$ , from a point on the infrastructural features to the farthest point in the study area is a little bit longer than the maximum buffer distance, i.e.  $D_{BFmax} > D_{Farthest}$ . Do not set the value of  $D_{BFmax}$  in such a way that  $D_{BFmax} < D_{Farthest}$ . No problem arises when  $D_{BFmax} > D_{Farthest}$ , because the rings outside the study area will be trimmed off.

Type [Number of rings] and [Distance between rings].  
Click [Next].

Then the dialog box shown in Figure 6.1.4 appears.

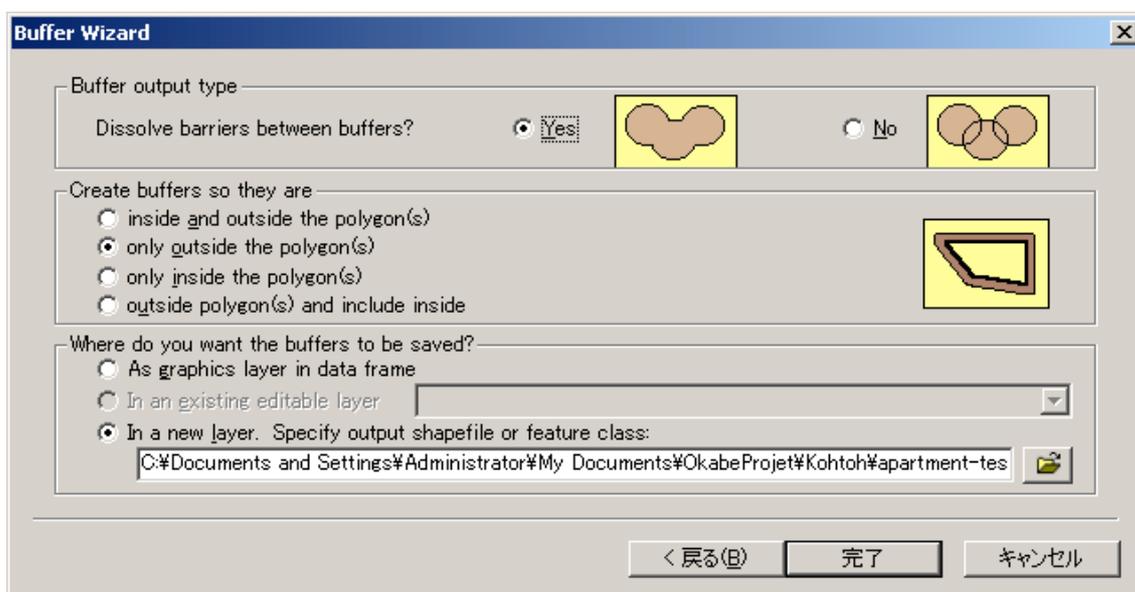


Figure 6.1.4

Click [Yes] for [Dissolve barriers between buffers?]

If the infrastructural features are polygons, check [only outside the polygon(s)] for [Create buffers so that they are].

Click [In a new layer.] for [Where do you want the buffers to be saved?], and type a new file name.

Click [Finish].

In a few minutes (or a few hours depending on the number of rings), the buffer rings are indicated by “contour lines” as in Figure 6.1.5.

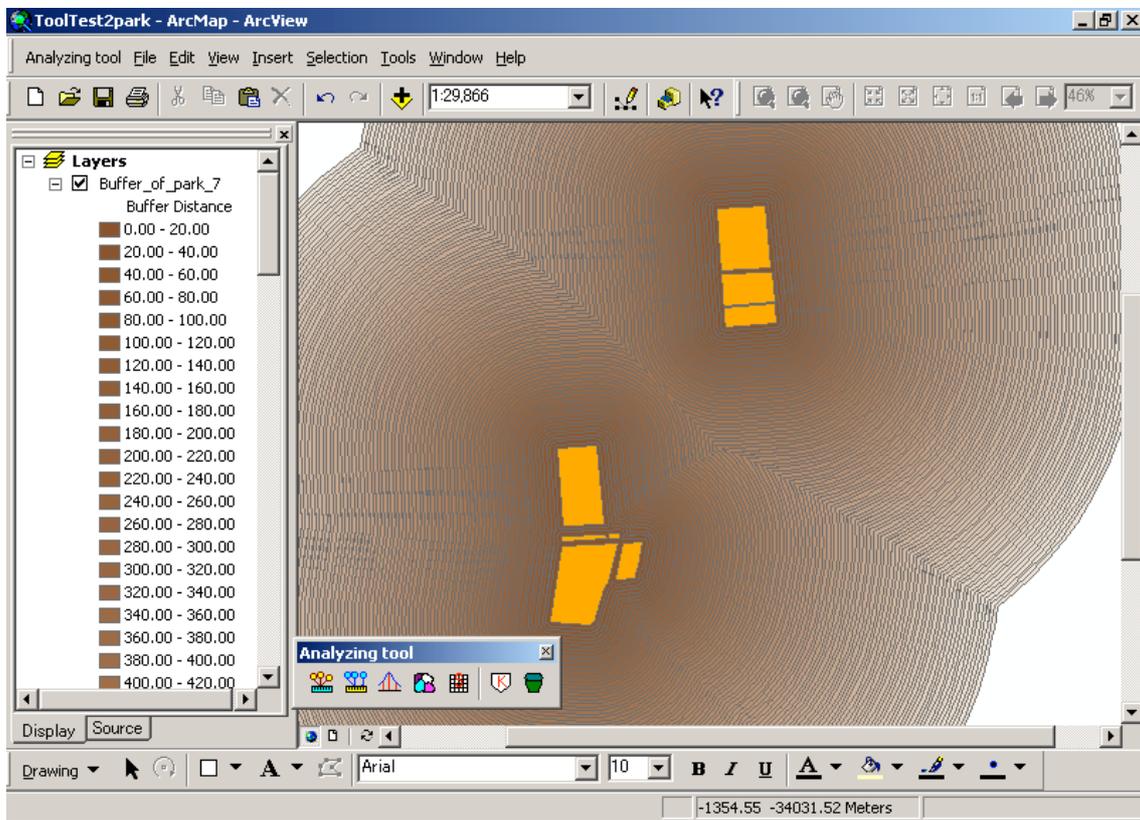


Figure 6.1.5

## 6.1.2 Trim off the buffer rings outside the study area

Click [Geoprocessing Wizard] in the [Tools] menu.

The dialog box shown in Figure 6.1.6 appears.

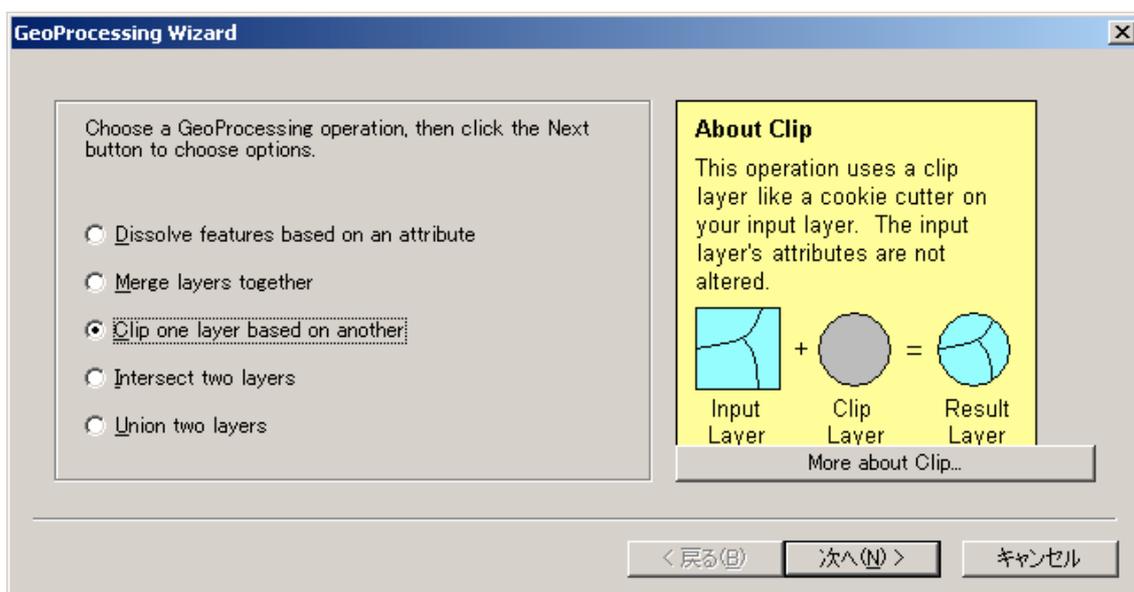


Figure 6.1.6

Click [Clip one layer base on another].

Click [Next].

The dialog box shown in Figure 6.1.7 appears.

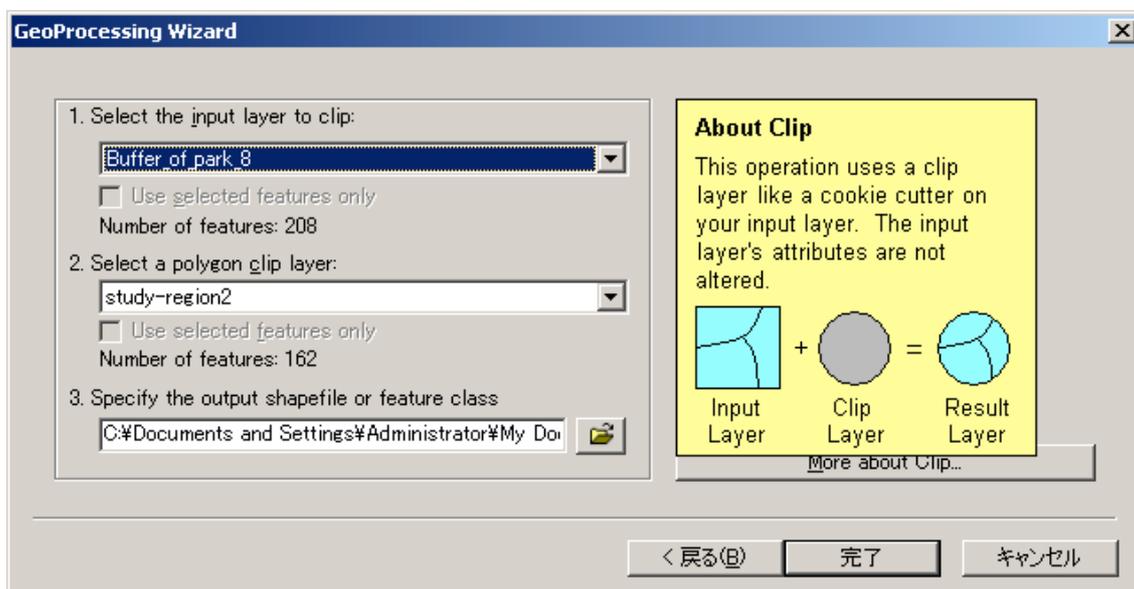


Figure 6.1.7

Using the pull-down menu of [Select the input layer to clip:], click the buffer layer generated in Section 6.1.1.

Using the pull-down menu of [Select a polygon clip layer], click our study region (e.g. the Kiba region in Tokyo).

Using the pull-down menu of [Specify the output shapefile or feature class], type a file name.

Click [Finish].

SAINF shows the buffer rings trimmed off the outside of the study region (the purple colored area in Figure 6.1.8) and the resulting trimmed buffer rings are stored in the above file.

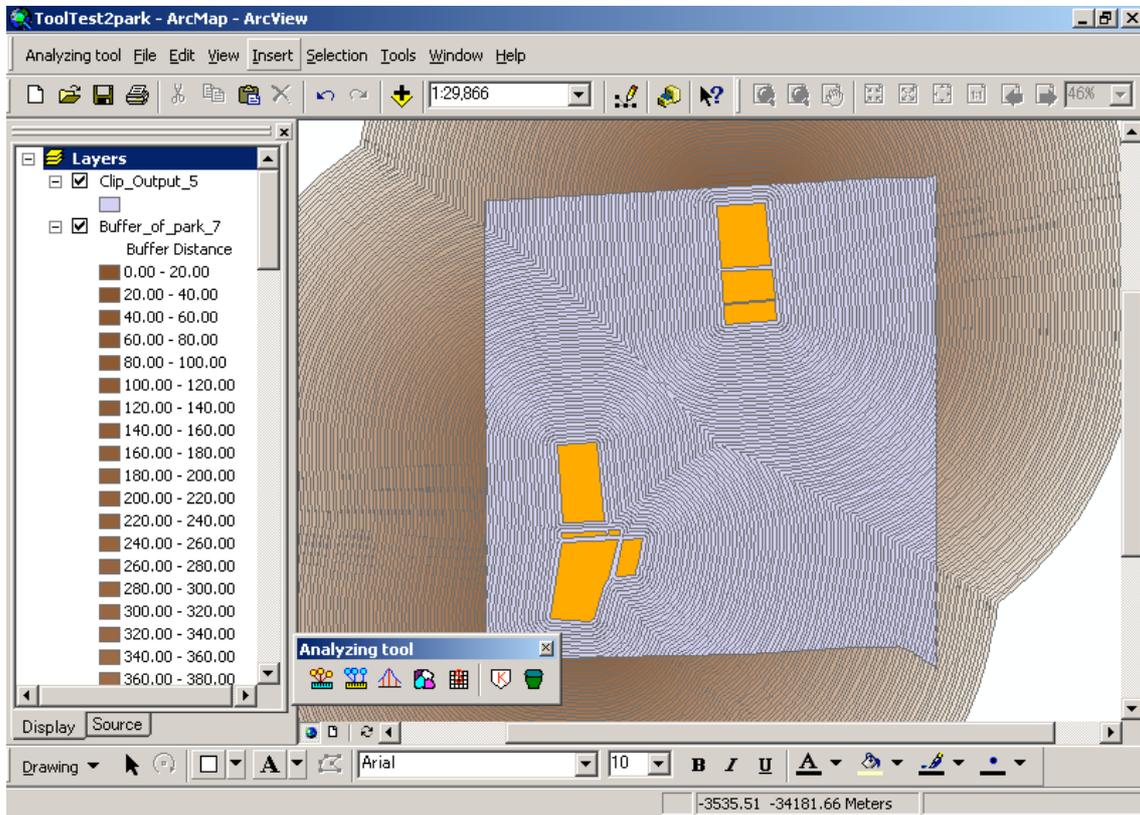


Figure 6.1.8

## 6.2 Calculation of the distance from each non-infrastructure feature (a point) to its nearest point on the infrastructural features

Open [SAINF Tools].

Click [NN Distance] (Figure 6.2.1).

Note that “NN Distance” means “Nearest Neighbor Distance”.

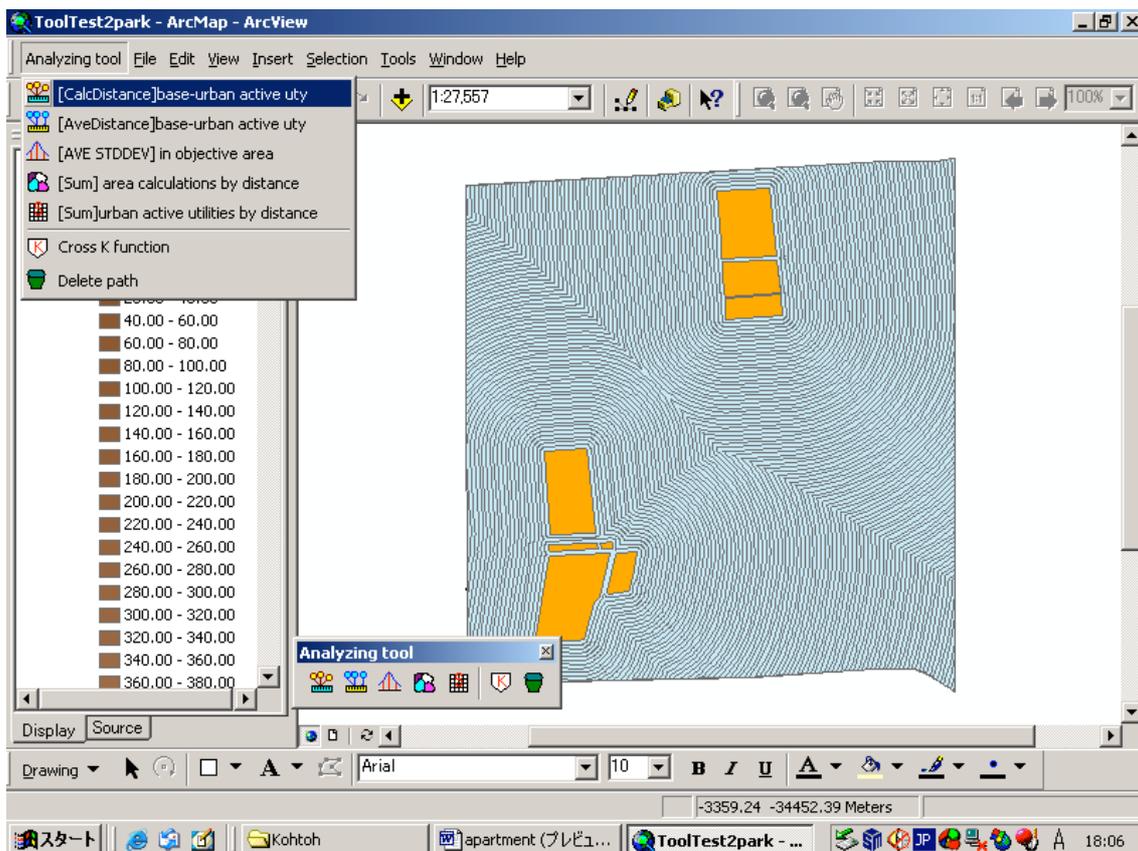


Figure 6.2.1

The dialog box [Infra feature layer] shown in Figure 6.2.2 appears.

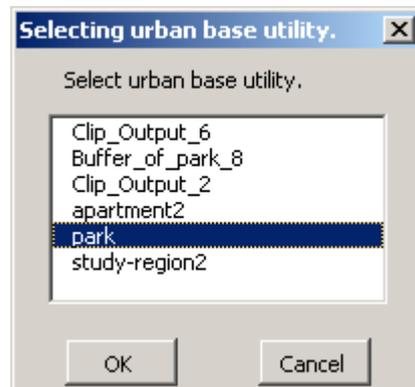


Figure 6.2.2

Click a layer of infrastructural features, e.g. the layer of parks in our example.  
Click [OK].

The dialog box [Non-infra feature layer] shown in Figure 6.2.3 appears.



Figure 6.2.3

Click a layer of non-infrastructural features, e.g. "apartment2" in our case.  
Click [OK].

The dialog box [Study region layer] shown in Figure 6.2.4 appears.

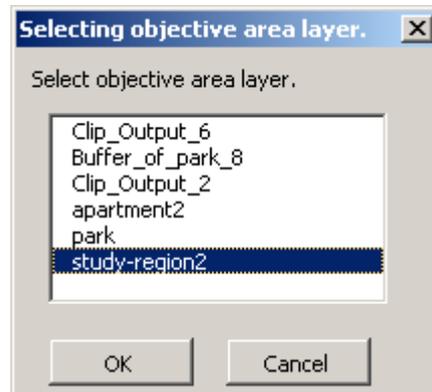


Figure 6.2.4

Click a study region, e.g. “study-region2” in our case.  
Click [OK].

The dialog box [Do you want to visualize the lines from non-infra features to their nearest infra features?] (Figure 6.2.5) appears.

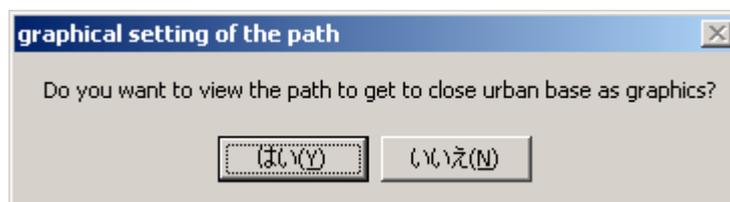


Figure 6.2.5

If you want to visualize the line from each non-infrastructure feature to its nearest infrastructure feature, click [Yes].

The lines are shown as in Figure 6.2.6.

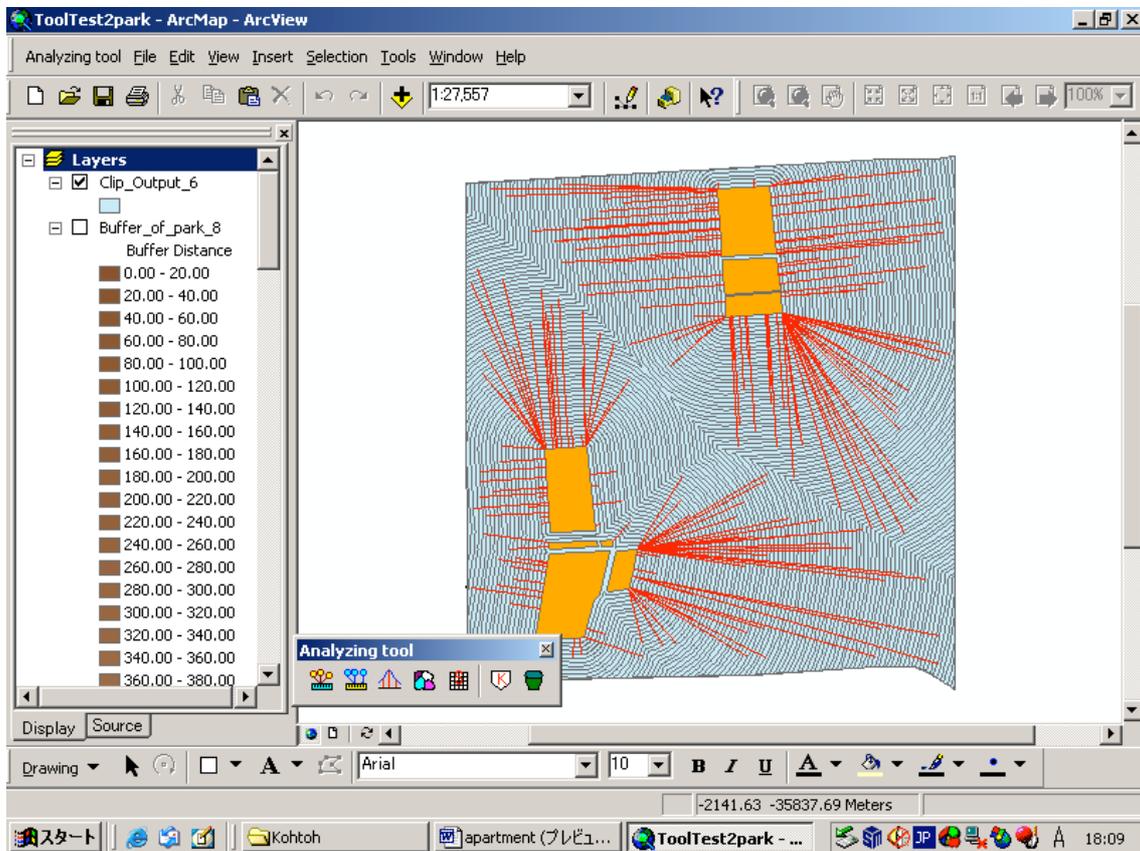


Figure 6.2.6

When computation is done, the field named [NN Distance] is added to the non-infrastructure feature layer (Figure 6.2.7).

FID	Shape*	Id	Distance
0	Point	0	1153.0
1	Point	0	1079.3
2	Point	0	983.1
3	Point	0	963.9
4	Point	0	856.4
5	Point	0	827.4
6	Point	0	820.1
7	Point	0	748.5
8	Point	0	750.6
9	Point	0	703.4
10	Point	0	592.0
11	Point	0	604.2
12	Point	0	613.5
13	Point	0	610.2
14	Point	0	611.1
15	Point	0	619.3
16	Point	0	492.9
17	Point	0	449.3
18	Point	0	484.8

Figure 6.2.7

### 6.3 Calculation of the average distance from non-infrastructure features to their nearest infrastructural features

Click [Average NN distance] in the [SAINF Tools] menu (Figure 6.3.1).

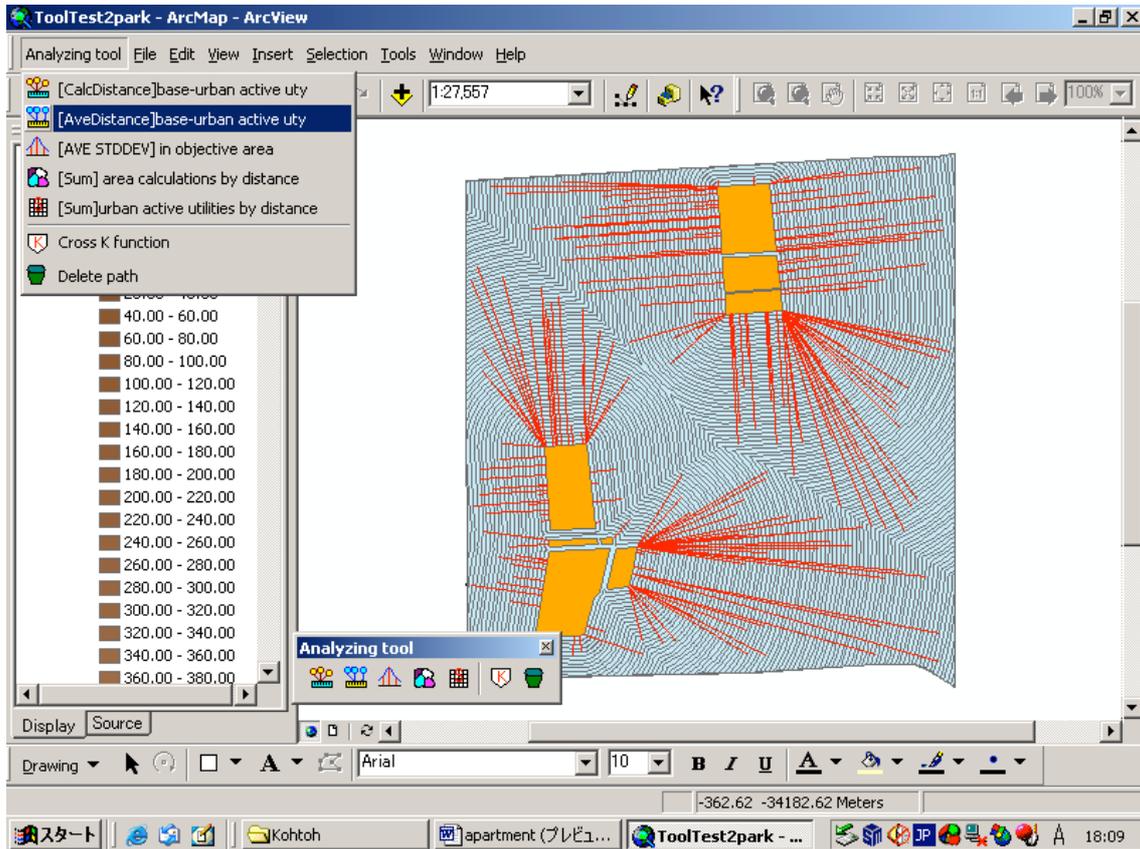


Figure 6.3.1

The dialog box [Non-infra feature layer] shown in Figure 6.3.2 appears.



Figure 6.3.2

Click one type of non-infrastructure features, e.g. parks in our case.  
Click [OK].

When the computation completes, the average nearest neighbor distance and the number of non-infrastructure features are shown (Figure 6.3.3). In our case, the former is 516.79, and the latter is 204. These numbers are denoted by  $\bar{d}$  and  $m$ , respectively.



Figure 6.3.3

## 6.4 The mean and the standard deviation of the distance from a random point to its nearest infrastructural feature

Click [Mean & Standard Deviation] in the [SAINF Tools] menu (Figure 6.4.1).

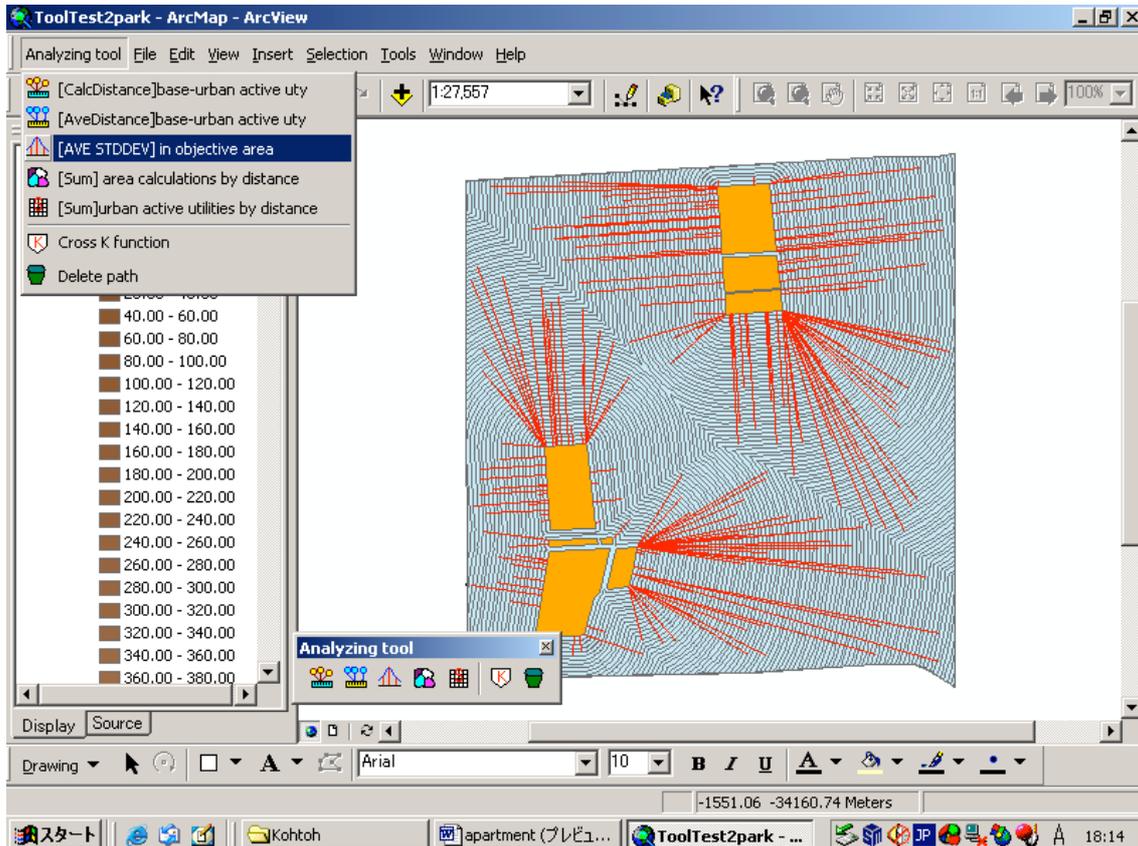


Figure 6.4.1

The dialog box [Select a buffer layer] shown in Figure 6.4.2 appears.

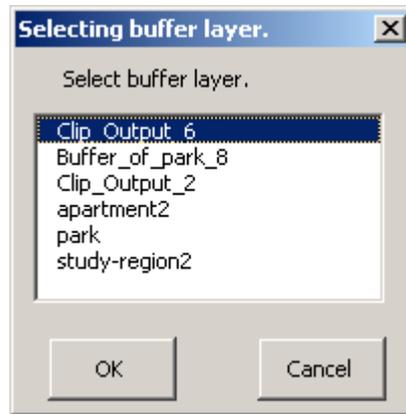


Figure 6.4.2

Click the layer created in Section 6.1.2 (in our case, “Clip Output 6”), and Click [OK]. Then the total area of the buffer rings, the longest buffer distance and the width of a buffer ring are shown (Figure 6.4.3).

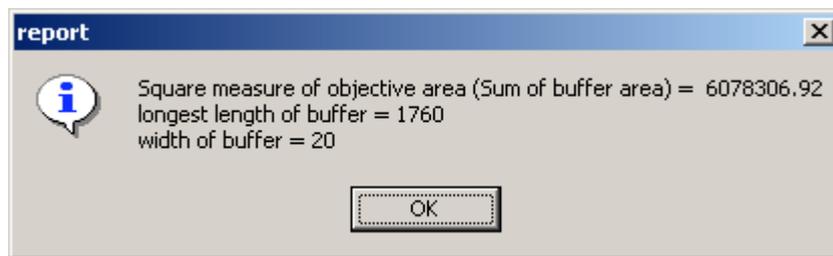


Figure 6.4.3

Click [OK] and then, in a few minutes of computation, SAINF gives the mean,  $\mu$ , and the standard deviation,  $\sigma$ , of the distance from a random point to the nearest point in infrastructural features (Figure 6.4.4). Note these figures on your memo pad. In our case,  $\mu=556.841$  and  $\sigma=375.811$ .

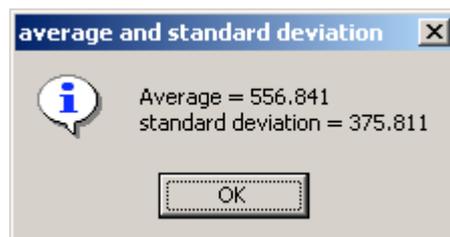


Figure 6.4.4

## 6.5 Analysis with the average nearest neighbor distance test

In Section 6.3, we have obtained the observed average nearest neighbor distance,  $\bar{d} = 516.79$  for  $m = 204$  non-infrastructure features. In Section 6.4, we have obtained the expected nearest neighbor distance,  $\mu = 556.841$ , and its standard deviation  $\sigma = 375.811$  under the null hypothesis  $H_0$  (i.e. non-infrastructure features are uniformly and randomly distributed over a study region). Comparing  $\bar{d} = 516.79$  with  $\mu = 556.841$ , we notice that apartment buildings (non-infrastructure features) tend to locate near parks in Kiba. To test this tendency statistically, we employ the mean test. To be explicit, the test statistic,  $z$ , is given by equation (2) in Section 2. In our case,  $z = -1.52216$ . Therefore the null hypothesis cannot be rejected with confidence level 95%. The above tendency is statistically weak.

## 6.6 Analysis with the goodness-of-fit test

An alternative test is the goodness-of-fit test. The method is shown in Section 2. This test is to compare the observed number of non-infrastructure features (e.g. high-class apartment buildings) in each interval with the expected number under the null hypothesis  $H_0$  (i.e. non-infrastructure features are uniformly and randomly distributed over a study region).

### 6.6.1 A file for ring intervals

Make a file that shows ring intervals (i.e.  $a_0, K, a_k$  in equation 3) using a text editor, such as Notepad in Windows. The first term  $a_0$  is fixed 0 and  $a_k$  should exceed the longest distance of the buffer rings obtained in Section 6.4. The width of each interval may be different, but the width should be the width fixed in Section 6.1 multiplied by a natural number. An example is:

---

0  
300  
600  
900  
1200  
1800

---

### 6.6.2 Calculation of the area of each buffer ring

When the null hypothesis  $H_0$  holds, the expected number of non-infrastructure features in a buffer ring is proportional to its area. To calculate this area, click [Areas of buffer rings] in the [SAINF Tools] menu (Figure 6.6.1).

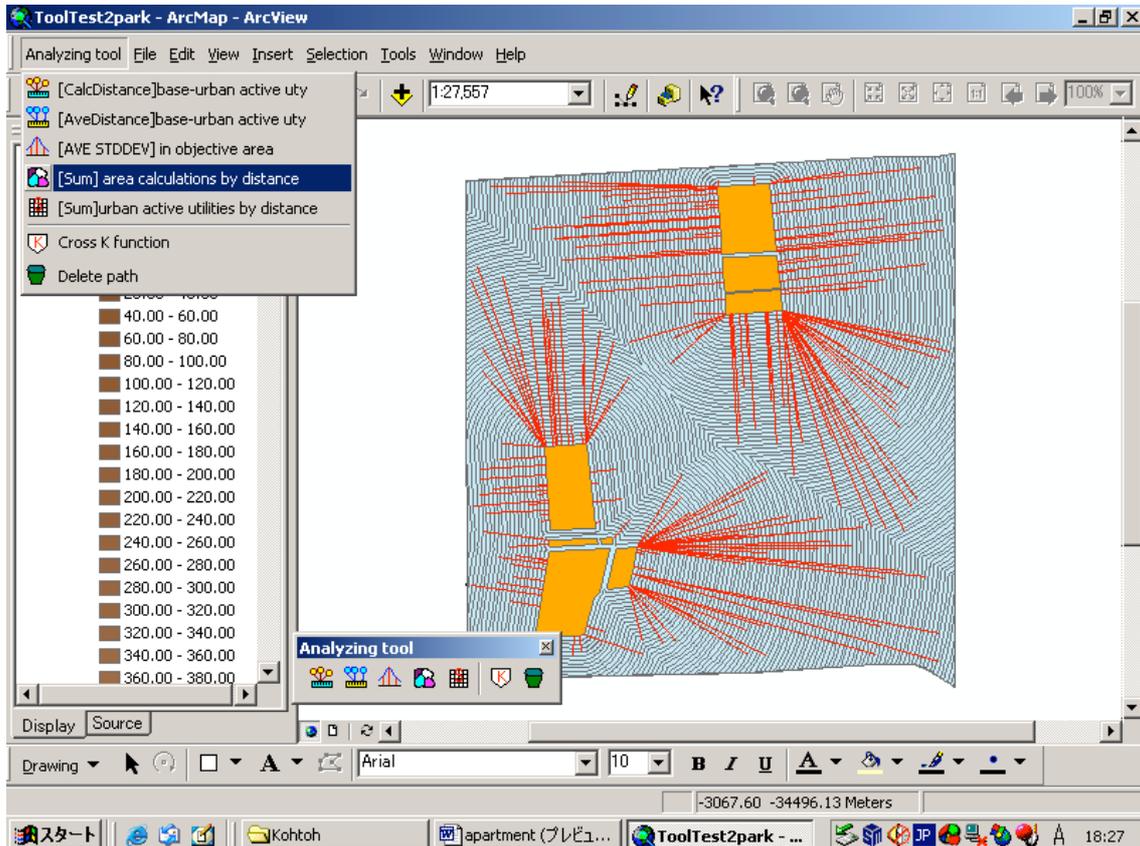


Figure 6.6.1

Then the dialog box shown in Figure 6.6.2 appears.

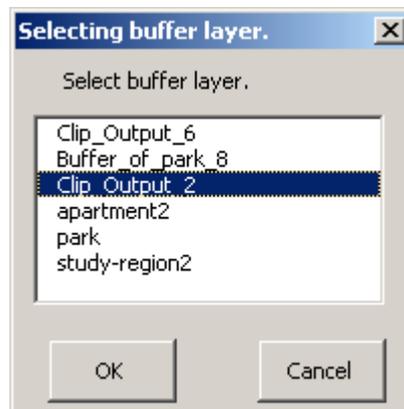


Figure 6.6.2

Click the layer made in Section 6.1.2 and click [OK].

Then the dialog box shown in Figure 6.1.3 appears.

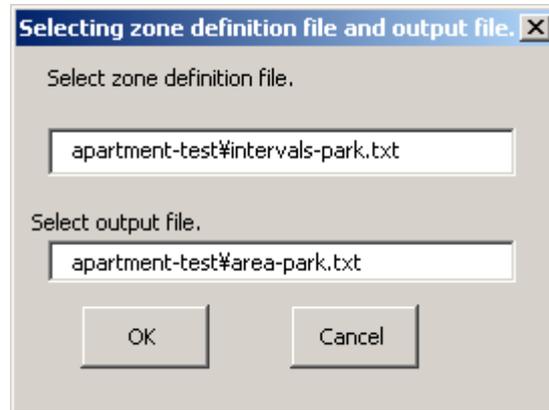


Figure 6.6.3

Type the file name made in Section 6.6.1 in the first row, type the name of a new file for storing the results in the second row, and click [OK].

When the computation is done, the results are written in the file as follows:

---

Notes: 0,100,S(1),12300 stands for that the area of S(1), in which X is larger than 0, and smaller than and equal to 100, is 12300.

0,300,S(1),1792534.951  
300,600,S(2),1817824.723  
600,900,S(3),1362014.769  
900,1200,S(4),690877.662  
1200,1800,S(5), 415054.818

The total amount of sum of area calculations = 6078306.923

---

### 6.6.3 Counting the observed number of non-infrastructurel features in each ring

Click [Number of non-infra features in rings] in the [SAINF Tools] menu (Figure 6.6.4).

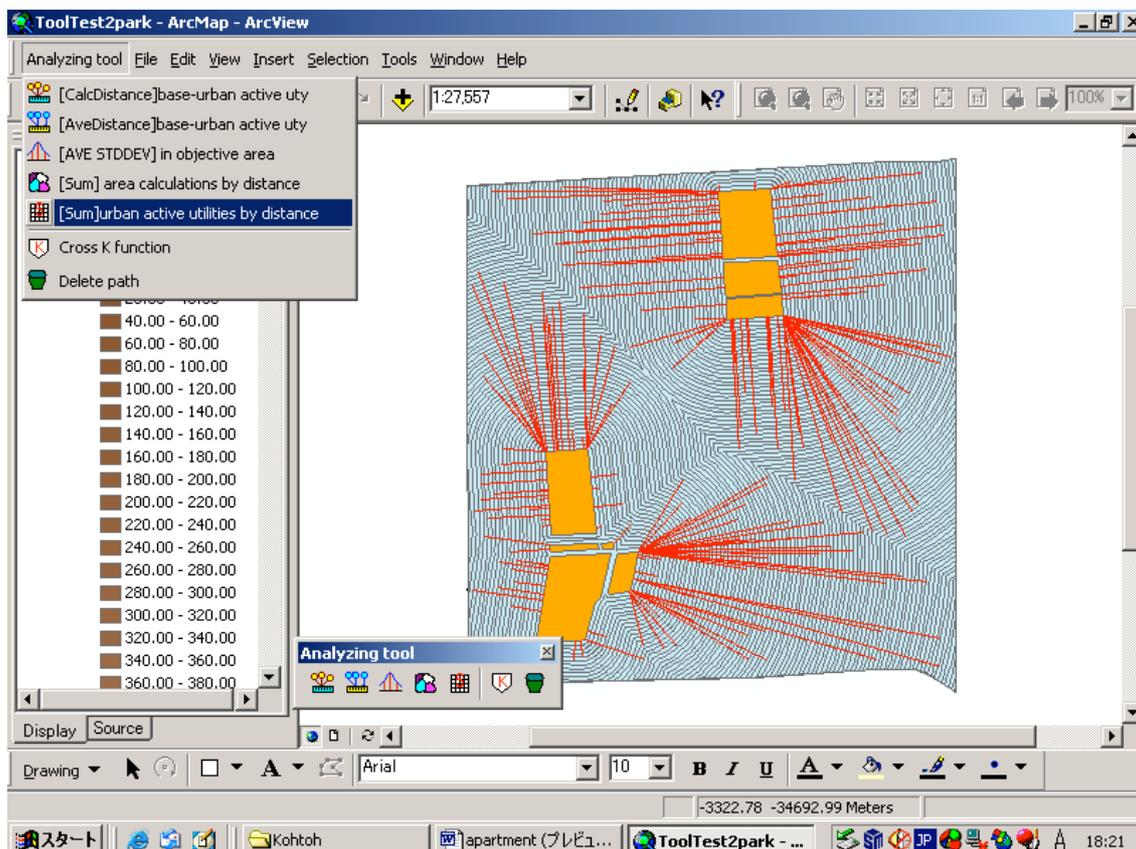


Figure 6.6.4

The dialog box shown in Figure 6.6.5 appears.

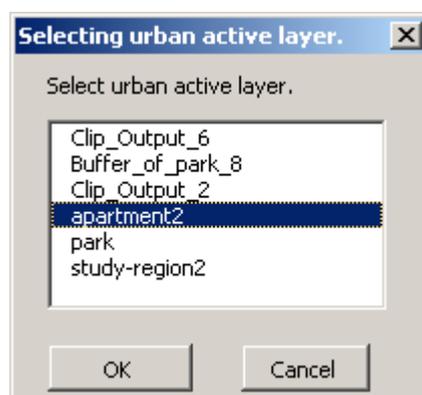


Figure 6.6.5

Click the layer of non-infrastructure features (e.g. high-class apartment buildings).  
Click [OK].

Then the dialog box shown in Figure 6.6.6 appears.



Figure 6.6.6

Click the layer of the study region (e.g. study-region2).  
Click [OK].

Then the dialog box shown in Figure 6.6.7 appears.

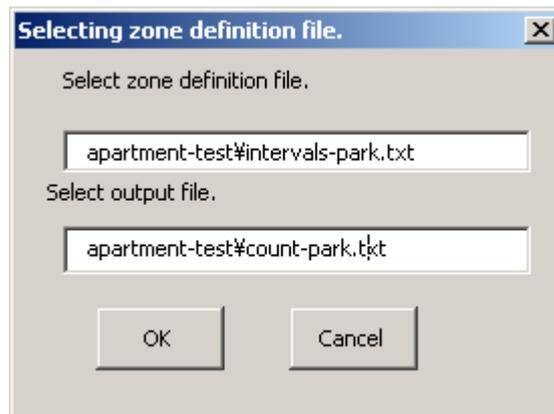


Figure 6.6.7

Type the name of the file made in Section 6.6.1 in the first row, type the name of a new file for the results in the second row, and click [OK].

When counting is done, the results are shown in the output file as follows:

---

Caution: 0,100,26 mean the number of urban activities, which are that the closest distance to get to base utilities is more than 0 and less than 100, is 26.

0,300,64

300,600,58

600,900,59

900,1200,12

1200,1800,11

The total amount of sum of the number of urban active utilities = 204/204

---

#### **6.6.4 Test with the goodness-of-fit**

Comparing the expected frequencies obtained from the areas of buffer rings in Section 6.6.2 and the observed frequencies obtained in Section 6.6.3, we can carry out the goodness-of-fit test.

An example is shown in Table 6.6.1 made by Microsoft EXCEL.

Table 6.6.1 Table for the goodness-of-fit

distance	area	expected	observed
0-300	1792534.951	60.16102	64
300-600	1817824.723	61.00979	58
600-900	1362014.769	45.71191	59
900-1200	690877.662	23.18722	12
1200-1800	415054.818	13.93006	11
total	6078306.923	204	204
chi square probability			0.036116

## 7. Cross *K*-function method

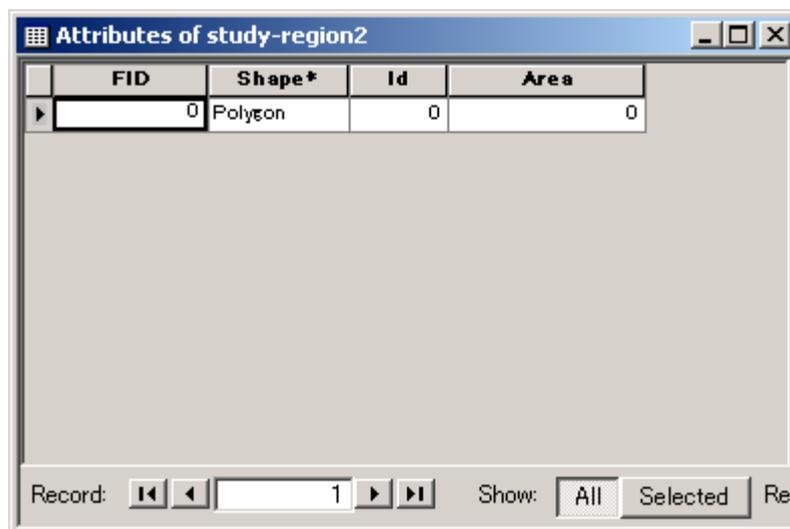
### 7.1 Cross *K*-function

The cross *K*-function is defined by equation (4) in Section 2. Detailed illustration of this function is found, for example, in Ripley.

### 7.2 General setting

The cross *K*-function in SAINF assumes:

- (1) infrastructural features are point-like features (e.g. stations in a city),
- (2) As is seen in Figure 7.2.1, the layer of a study region includes the field named “Area”. If the layer does not include it, open the attribute table of the layer of the study region and add the field named “Area” by clicking [option] and [add field].



FID	Shape*	Id	Area
0	Polygon	0	0

Figure 7.2.1

### 7.3 Estimation of the cross K-function

Click [Cross K-function] in the [SAINF Tools] (Figure 7.3.1).

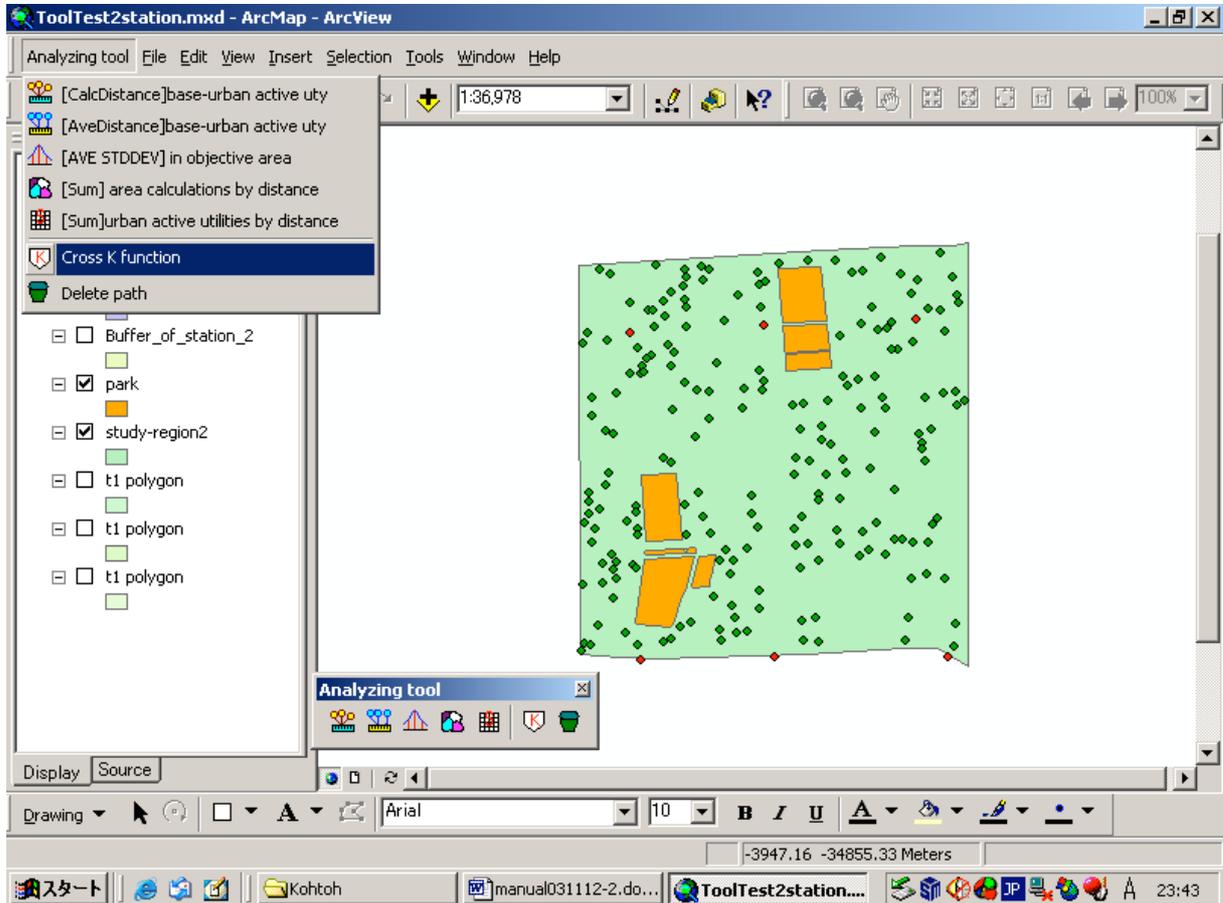


Figure 7.3.1

The dialog box shown in Figure 7.3.2 appears. Click a layer of infrastructural features, for example, “stations”, and click [OK].

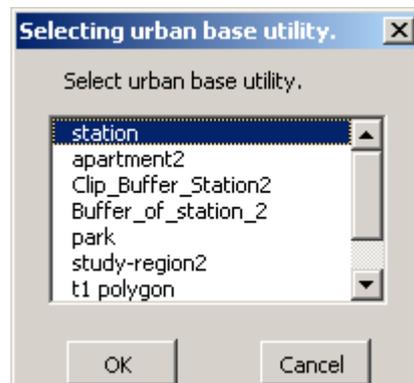


Figure 7.3.2

Then the dialog box shown in Figure 7.3.3 appears.

Click a layer of non-infrastructure features, for example, “apartment2” (implying high-class apartment buildings), and click [OK].

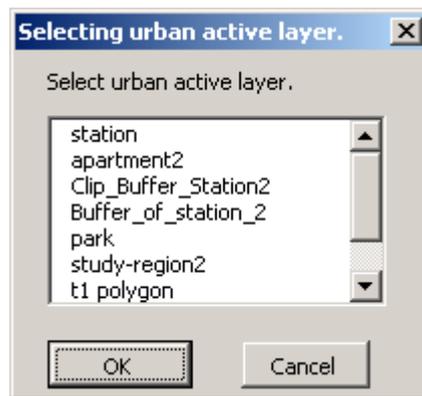


Figure 7.3.3

Then the dialog box shown in Figure 7.3.4 appears. Click a layer of a study region, for example, “study-region2”, and click [OK]. If this layer does not contain the field named “Area”, an error message appears. See Section 7.2.

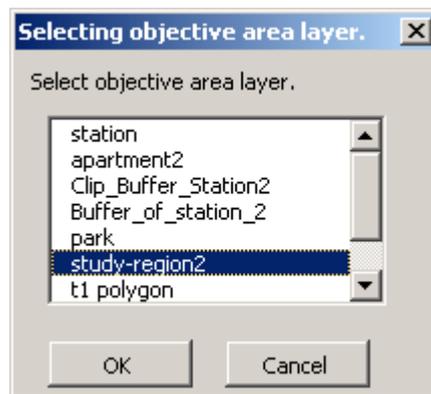


Figure 7.3.4

Then the dialog box shown in Figure 7.3.5 appears. Type a new file name for output and click [OK].

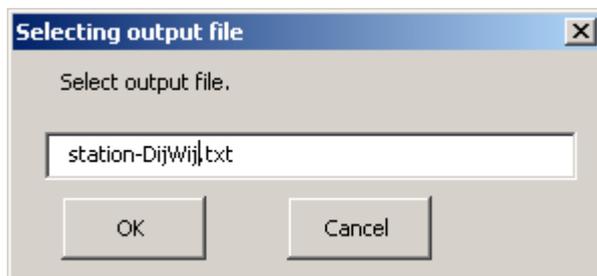


Figure 7.3.5

Usually it takes twenty minutes or so for computing the  $K$ -function. When the computation is done, the message shown in Figure 7.3.6 appears.

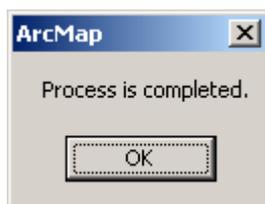


Figure 7.3.6

The output file looks like Table 7.3.1.

Table 7.3.1

---

Start = 23:56:56		
N1 = 3	N2 = 204	Area = 6509704.292
Dij, Wij		
1125.439,0.635		
1043.475,0.647		
1135.61,0.634		
873.562,0.678		
756.154,0.71		
727.367,0.72		
706.081,0.728		
640.655,0.758		
...		
1665.924,0.335		
2043.427,0.32		
2094.869,0.318		
2043.987,0.32		
1910.568,0.325		
End = 23:57:01		

---

In this table, the following notation is used:  $N_1$  is the number of the infrastructural features;  $N_2$  is the number of the non-infrastructural features;  $D_{ij}$  is the distance from the  $i$ th infrastructural feature to the  $j$ th non-infrastructural feature;  $W_{ij}$  is the proportion of the circumference of the circle centered on the  $i$ th infrastructural feature with  $D_{ij}$  radius which is within the study region.

In order to take the boundary effect into account, the observed cross  $K$ -function given by equation (4) need be modified (Bailey and Gatrell 1995). The observed cross  $K$ -function considering the boundary effect exactly is given by

$$K(t) = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \frac{I(t, D_{ij})}{w_{ij}}, \quad (5)$$

where  $I(t, D_{ij})$  is 1 if  $D_{ij} \leq t$  and 0 otherwise. The exact expected cross  $K$ -function

under the null hypothesis  $H_0$ ,  $E(K(t))$ , is given by

$$E(K(t)) = \pi^2. \quad (6)$$

By comparing these two functions, we can test the null hypothesis  $H_0$ .

## References

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